

Performance Differences on a Measure of Mathematics Vocabulary
for English Learners and Non-English Learners With and Without Mathematics Difficulty

Sarah R. Powell, Katherine A. Berry, and Le M. Tran

The University of Texas at Austin

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Author Note

Sarah R. Powell, Department of Special Education, The University of Texas at Austin; Katherine A. Berry, Department of Special Education, The University of Texas at Austin; Marcia A. Barnes, Department of Special Education, Vanderbilt University.

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Correspondence concerning this article should be addressed to Sarah R. Powell, 1 University Station D5300, Austin, TX 78712. E-mail: srpowell@austin.utexas.edu

Abstract

The purpose of this study was to measure the mathematics-vocabulary performance of 3rd graders to determine if performance differences existed among English learners (ELs) and non-ELs with and without mathematics difficulty (MD). Using screening data from a large-scale intervention study, we categorized students into four groups based on equation solving and word-problem performance: no MD, equation difficulty, word-problem difficulty, or equation and word-problem difficulty. Within these four categories, we compared student performance between ELs and non-ELs. Results showed ELs scored lower than non-ELs across all groups. Both ELs and non-ELs with equation and word-problem difficulty demonstrated lower performance on the mathematics vocabulary measure than students with only equation difficulty or word-problem difficulty. Findings indicated ELs and students with MD have limited mathematics vocabulary knowledge and may require focused instruction on the language of mathematics.

Keywords: English learners; learning difficulties; mathematics; vocabulary; word problems

Performance Differences on Mathematics Vocabulary

for English Learners and Non-English Learners With and Without Mathematics Difficulty

Developing mathematics vocabulary competency is an essential skill beginning in the early elementary grades as students are required to speak, listen to, and read mathematics vocabulary presented within text. Therefore, understanding and communicating effectively in mathematics requires a comprehensive knowledge of mathematics vocabulary. English Learners (ELs) and students with mathematics difficulty (MD) represent two subgroups of students who struggle to acquire skill with the language of mathematics (Forsyth & Powell, 2017; Orosco, 2014). The purpose of this study was to investigate the mathematics-vocabulary performance of third-grade students, ELs and non-ELs with and without MD, to determine if mathematics-vocabulary knowledge differed among these student groups.

This introduction includes a description of the language of mathematics and explores why mathematics vocabulary proves especially difficult for students. First, we discuss ELs and their mathematics performance. Then, we present an explanation of different types of MD and typical performance for students with MD. Within the descriptions of both populations, we highlight specific challenges related to mathematics vocabulary. Finally, we identify ELs with MD as at-risk due to challenges in both mathematics and English, and we present the purpose and research questions of this study.

Language of Mathematics

In school, students learn academic language, including vocabulary, grammatical structures, and linguistic functions (Cummins, 2000). Academic language promotes engagement in activities and learning in the content areas (Schleppegrell, 2012). Integral to academic language is domain-specific academic vocabulary (Baumann & Graves, 2010), which serves as a

prerequisite for understanding ideas, concepts, and procedures within a content area. In this study, we focus on academic vocabulary in the domain of mathematics.

Mathematics vocabulary may present challenges for many students (Forsyth & Powell, 2017; Powell & Nelson, 2017; Powell, Driver, Roberts, & Fall, 2017). The sheer volume of mathematics-vocabulary terms complicates the learning of mathematics vocabulary (Hughes, Powell, & Lee, in press; Smith & Angotti, 2012), but so does the complexity of mathematics-vocabulary terms. Students may experience difficulty because mathematics-vocabulary terms (a) may have a shared meaning in mathematics and general English, (b) may have different meanings in mathematics and general English, (c) may have multiple meanings in mathematics, (d) may have a shared meaning with another content area, (e) may be homonyms with other terms, or (f) may be confused with terms from another language (Rubenstein & Thompson, 2002). Teachers also may use informal mathematics vocabulary and not provide enough opportunities for students to use formal mathematics vocabulary (Livers & Elmore, 2018), which could limit the learning of mathematics vocabulary for students. In many ways, every student in a mathematics classroom is a language learner (Barrow, 2014), and a need exists to understand the specific barriers to mathematics for different student populations.

ELs and Mathematics Performance

The term EL, or English Learner, refers to students whose native language is a language other than English. ELs represent a diverse group of students from various linguistic, cultural, and educational backgrounds. Comprising 10% of the United States student population (National Center for Education Statistics, 2017), ELs have been the fastest growing subgroup for the past 20 years (Klingner & Eppolito, 2014). By 2030, ELs are projected to represent 40% of all school-age students (U.S. Department of Education, National Center for Education Statistics

[NCES], 2013). Notably, many ELs demonstrate limited English proficiency and struggle to meet the academic demands in classrooms where the primary language is English (Driver & Powell, 2017; Liguanti & Cook, 2013).

Accessing English language proves especially challenging for ELs in the mathematics classroom due to the high linguistic demands of the subject. In elementary mathematics classes, students must understand and solve problems and explain their methods for problem solving in verbal and written forms (Moschkovich, 1999; Powell & Hebert, 2016). The expectation to justify mathematical answers through language reflects a central component of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, the requirement to use explanations to communicate mathematical thinking places added academic pressure on ELs, who are not native English speakers (Moschkovich, 2015). As a result, ELs often yield lower mathematics scores on national assessments than their non-EL peers, with challenges related to the concepts and language of mathematics contributing to lower performance (Abedi & Lord, 2001; Wolf & Leon, 2009). The 2017 National Assessment for Education Progress (NAEP) data revealed that 86% and 94% of fourth- and eighth-grade ELs scored below proficient in mathematics (NCES, 2017). The mathematics achievement gap between ELs and non-ELs starts early, doubles from fourth to eighth grade, and leads to detrimental effects in high school and beyond, including increased risk for school failure, lower graduation rates, and unemployment (Menken, 2013; NCES, 2017; Randel, Moore, & Blair, 2008; Reardon & Galindo, 2009).

Mathematics vocabulary. Although ELs may perform comparable to their non-EL peers on mathematics tasks involving basic computation skills, ELs experience challenges understanding and interpreting the language of mathematics (Orosco, 2014). Mathematics

vocabulary plays a critical role in ELs' mathematics performance due to the technical definitions, multiple representations and meanings, overlap with everyday English terms, homonyms, and similarity to native language terms (Roberts & Truxaw, 2013).

Mathematics vocabulary includes technical definitions, symbols, and diagrams unfamiliar to most ELs. For example, the terms *sum*, *value*, and *product* have specific and complex mathematical definitions familiar to most native speakers, but new to ELs with limited knowledge of terms (Freeman & Crawford, 2008). Symbolic vocabulary terms such as *zero* and *equal*, which are used to explain numerals and symbols, often are unknown to ELs (Powell et al., 2017). Mathematics vocabulary concepts also may be represented in multiple ways and present with multiple meanings, or the same term may be used to describe more than one situation. For example, over 10 different terms exist to describe subtraction; a *quarter* may refer to a coin or a fourth of a whole (Moschkovich, 2002). The overlap between mathematics vocabulary and everyday English terms represents another area of difficulty for ELs. The term *volume*, for instance, refers to the amount of space in mathematics but describes a noise level in everyday English. Similarly, *cubed* means raised to the third power in mathematics but explains a way to cut vegetables in everyday English. Mathematics-vocabulary terms become increasingly confusing with the introduction of homonyms like *whole/hole*, *half/have*, and *symbol/cymbal*, to name a few. Lastly, many English vocabulary terms are similar to native language terms, which may cause misunderstanding for ELs. Albeit sometimes helpful, similarities also may contribute to confusion for ELs. For example, the Spanish term for *quarter* is *cuarto*, which can mean quarter of an hour, but *quarter* also refers to a room in a house as in living quarters (Roberts & Truxaw, 2013). Ultimately, mathematics vocabulary represents a distinct area of difficulty for ELs due to the “double demands” (Baker et al., 2014) of simultaneously acquiring proficiency in

English and the language of mathematics. To succeed in the mathematics classroom, ELs must learn to navigate between the English and mathematics-language registers, where the meaning of terms often changes between the two (Cirillo, Richardson Bruna, & Herbel-Eisenmann, 2010; Jourdain & Sharma, 2016; Roberts & Truxaw, 2013).

Vocabulary knowledge serves as a prerequisite skill for solving most mathematics problems but especially for solving word problems (Chan, 2015). Word problems include a combination of words and numbers that require interpretation by the student (e.g., Laura has 15 tacos. Emily has 32 tacos. How many more tacos does Emily have?) and comprise the majority of items on high-stakes assessments. Solving word problems requires an extensive academic and mathematics vocabulary, which is lacking in many ELs who present with low English proficiency. With mathematics vocabulary deficiencies and the high linguistic demands of word problems, ELs will struggle to develop word problem proficiency beginning as early as kindergarten (Morgan, Farkas, & Wu, 2009). In the following section, we describe another population of students who experience difficulty with mathematics vocabulary: students with MD.

Students with MD

Students identified with learning disabilities in mathematics, sometimes referred to as dyscalculia (e.g., Butterworth, 2010), account for approximately 3 to 6% of all school-age students (Shalev et al., 2000). The term MD is frequently used in the field of special education because the term includes students who receive special education services and those who experience mathematics difficulty without a formal disability diagnosis (Fuchs, Fuchs, & Compton, 2012; Szűcs & Goswami 2013). Similarly, we use the umbrella term MD to describe both students with a diagnosed disability and undiagnosed students who experience persistent

low mathematics performance.

Students with MD regularly demonstrate lower mathematics performance than their typically developing peers without MD, specifically with counting (Geary et al., 2000), arithmetic (Tolar et al., 2016), whole-number computation (Raghubar et al., 2009), comparison (De Smedt & Gilmore, 2011), word-problem solving (Krawec et al., 2012), rational-number understanding (Fuchs et al., 2013), and algebra (O'Shea et al., 2016). As a result of these challenges, students with MD are at great risk for school failure (Wei, Lenz, & Blackorby, 2013). For example, 70% of children who score below the 10th percentile in mathematics at the end of kindergarten receive a diagnosis of MD by fifth grade (Morgan et al., 2009), and over 95% of students with MD in fifth grade continue to demonstrate performance below the 25th percentile in high school (Shalev, Manor, & Gross-Tsur, 2005), indicating the persistence of disability.

Approximately 30 to 70% of students with MD present with comorbid difficulty in reading (Willcutt et al., 2013). Importantly, students with MD who also experience reading difficulty often demonstrate lower scores on a range of mathematics tasks including fact fluency, computation, place value, and word-problem solving (Andersson, 2010; Jordan, Hanich, & Kaplan, 2003; Jordan, Wylie, & Mulhern, 2010) when compared to students who experience MD without reading difficulty. Researchers often describe phonological processing and verbal comprehension as contributing factors to the differences between students with MD with and without reading difficulty (Child, Cirino, Willcutt, & Fuchs, 2019; Peterson et al., 2017; Slot et al., 2016). Similarly, the reading and language performance of students with MD comorbid with reading difficulty may contribute to lower performance on mathematics tasks (Raddatz, Kuhn, Holling, Moll, & Dobel, 2017).

Categories of MD. Students with MD represent a heterogeneous disability population

because they are likely to present with deficits in several areas (Andersson, 2010). For example, students with MD may have weak number sense and struggle to retrieve mathematics facts, which increases the number and frequency of their computational and procedural errors when solving equations (Geary, 1993; Nelson & Powell, 2018b). In this study, we focus on students with MD who experience difficulty with solving equations. This focus on equations aligns with one of the two mathematics areas for specific learning disability (i.e., mathematics calculation) outlined by the Individuals with Disabilities Education Act (2004) in the United States.

Students with MD may also experience challenges with problem solving, especially when presented with word problems (Powell, Fuchs, et al., 2015). Word problems often require students to decode a written narrative, create a problem model, follow multiple steps to develop a problem solution, number a graph, and identify irrelevant or missing information (Fuchs & Fuchs, 2002). The multi-step nature of word problems may overwhelm students with MD, who tend to lack simple mathematics fluency skills and may experience comorbid difficulty with reading (Jordan et al., 2010). In this study, we also focus on students with MD who demonstrate difficulty with solving word problems, which aligns with the other mathematics area of specific learning disability (i.e., mathematics problem solving) designated by the Individuals with Disabilities Education Act (2004).

The language of math. An important area of concern for students with MD in the mathematics classroom relates to the language of mathematics. Specifically, students with MD may struggle with mathematics due to weakness in skills related to reading and language. Phonological processing, for example, correlates to students' retrieval skills of basic math facts, and the quality of phonological codes in long-term memory affects overall arithmetic performance (De Smedt, Taylor, Archibald, & Ansari, 2010). A lack of phonological awareness

leads to impairments in the areas of decoding, fluency, and comprehension; such deficits create difficulty with solving mathematics word problems (De Smedt et al., 2010).

The role of mathematics vocabulary also may contribute to lower mathematics performance for students with MD. Compared to students without MD, students with MD have limited knowledge of mathematics vocabulary presented in questions on assessments (Wieher, 2010). Students with MD regularly demonstrate weaker understanding of mathematics vocabulary (Powell et al., 2017) with students with comorbid MD and reading difficulty exhibiting the lowest performance on mathematics-vocabulary measures (Forsyth & Powell, 2017). The number and complexity of mathematics-vocabulary terms often creates confusion for students with MD, who lack the mathematics-vocabulary knowledge of students without MD (Schleppegrell, 2007).

ELs with MD may represent a subpopulation of students at risk due to their challenges with complex mathematical vocabulary and problem solving (Doabler, Nelson, & Clarke, 2016). A significant need exists to determine these students' baseline mathematics-vocabulary knowledge to better inform mathematics instruction, particularly in word-problem solving. However, to date, there is a paucity of empirical research that examines the mathematics vocabulary performance of elementary-aged EL and non-EL students with and without MD. To address this aim, we developed the current study to determine if performance differences existed among third-grade ELs and non-ELs with and without MD on a measure of mathematics vocabulary.

Purpose and Research Questions

To understand how third-grade students, both ELs and non-ELS, understand the language of mathematics, we administered a measure of mathematics vocabulary. We

categorized students into four groups based on equation solving and word-problem performance (i.e., no MD, equation difficulty, word-problem difficulty, or equation and word-problem difficulty) to identify mathematics-vocabulary differences based on MD status. We asked the following research questions:

1. Do mathematics-vocabulary performance differences exist between ELs and non-ELs with and without MD in the four difficulty categories?
2. Do mathematics-vocabulary performance differences exist among the four difficulty categories for ELs and non-ELs with and without MD?
3. Which mathematics-vocabulary terms cause the greatest difficulty for students?

Method

Context and Setting

After we received approval from our university's Institutional Review Board and our participating school district, we recruited elementary schools from a large urban school district in the Southwest of the United States. The school district served over 80,000 students. In 2017, the district reported 55.5% of students as Hispanic, 29.6% as Caucasian, 7.1% as African American, and 7.7% as belonging to another racial or ethnic category. In the district, 27.1% of students qualified as English learners, and 12.1% received special education services. Overall, 52.4% of students qualified as economically disadvantaged.

Participants

We recruited third-grade teachers for study participation across two school years: 2016-2017 and 2017-2018. During 2016-2017, we worked in 53 classrooms with 37 teachers representing 13 schools. In the 2017-2018 school year, we recruited 44 teachers from 13 schools teaching 51 classrooms. Several schools used departmentalization (i.e., the same teacher taught

multiple mathematics classes), which accounted for the difference in the numbers of teachers and classrooms. Across two years, we recruited 1,734 students.

From our original sample of 1,734 students, we asked teachers from all 104 classrooms to complete demographic information on the students in their classrooms from whom they received parental consent. However, some teachers did not provide demographic information for the students in their classroom or only provided demographic information for a select group of students with MD who participated in an efficacy trial about word-problem interventions. If teachers did not provide any demographic data for their classrooms, we were unable to include their classrooms in the analysis. As a result, 476 students from our original sample of 1,734 were not included in this analysis. Across the two cohorts, teachers from 92 classrooms provided demographic information on the consented students within the classrooms. Importantly, in these 92 classrooms, teachers received consent from between 8 to 16 students. We used our final sample of consented students ($n = 1,258$) with complete demographic information to conduct the analysis for this study.

We categorized students into four groups based on equation solving and word-problem performance: no mathematics difficulty, equation difficulty (EQD; i.e., performance below 27th percentile), word-problem difficulty (WPD; i.e., performance below 28th percentile), or word-problem and equation difficulty (EQD+WPD). We selected equation difficulty and word-problem difficulty as categories based on their alignment with the two mathematics areas identified for a specific learning disability diagnosis within the Individuals with Disabilities Education Act (2004): mathematics calculation (i.e., equation solving) and mathematics problem solving (i.e., word-problem solving). We aimed to categorize students with difficulty as performing below the 25th percentile, which is a commonly used cut-score in MD research

(Nelson & Powell, 2018a), and the 27th and 28th percentiles represent the closest percentiles (to the 25th) for which we could easily identify whole-number cut-scores.

Within these four categories, we compared student performance across EL and non-EL status. Classroom teachers provided the information about EL status including ratings on the Texas English Language Proficiency System (TELPAS) in which a score of 1 = beginning, 2 = intermediate, 3 = advanced, and 4 = advanced high. Teachers accessed students' TELPAS scores, which are currently the only method used to determine progress of ELs, using a secure online school portal for authorized personnel only. Table 1 provides sample sizes for each category based on EL status, demographic information, TELPAS scores, and means and standard deviations for the sample.

Mathematics Instruction for Students with MD

All students participated in mathematics instruction provided by their general education teacher. In the district, teachers primarily used *GO Math!*, *Houghton Mifflin Math*, and *enVisionMATH* curricula to guide mathematics instruction.

Measures

Word-problem solving. We used a measure of Single-Digit Word Problems to understand word-problem performance (Jordan & Hanich, 2000). We administered this measure as the first in the whole-class pretesting session. Single-Digit Word Problems included 14 one-step word problems involving sums or minuends of 9 or less categorized into the Total, Difference, and Change schemas. Interventionists read each word problem aloud and could re-read each problem up to one time upon student request. We scored Single-Digit Word Problems as the number of correct responses (maximum = 14). We calculated Cronbach's α on this sample as .88. We noted a significant correlation ($r = .534$) between Single-Digit Word Problems and a

measure of Double-Digit Word Problems (Powell & Berry, 2015).

Equation solving. The second measure administered in the whole-class pretesting session was Open Equations (Powell, 2007). For Open Equations, students solved 10 equations in a standard (e.g., $3 + __ = 8$) format. Students also solved equations in nonstandard formats, including two identity statements (e.g., $__ = 4$), 10 nonstandard equations with an operator symbol on the right side (e.g., $5 = 9 - __$), and eight nonstandard equations with operator symbols on both sides (e.g., $9 - 6 = 7 - __$). Excluding the identity statements, 14 of the equations included addition operator symbols and 14 included subtraction operator symbols. Students completed as many problems as possible within the 6 min timing. We scored the measure as the number of correct answers, with a maximum score of 30. Cronbach's α for this sample was .93. We identified a significant correlation ($r = .618$) between Open Equations and Equivalence Problems (Powell & Driver, 2012), another measure of nonstandard equation solving.

Mathematics vocabulary. In the whole-class pretesting session, we also administered Mathematics Vocabulary: Grade 3 (Powell & Tran, 2016). With this measure, we intended to capture students' mathematics vocabulary understanding. To develop Mathematics Vocabulary: Grade 3, we revised and condensed a previous measure of mathematics vocabulary designed for use across grades 3, 4, and 5 (Powell et al., 2017). We created the initial measure of mathematics vocabulary (Powell et al., 2017) by compiling an inventory of the common mathematics vocabulary terms introduced at each grade level. Specifically, we examined the glossaries in *GO Math!*, *enVisionMATH*, and *Everyday Mathematics* because of the widespread adoption of these curricula across the United States. After developing a list of all of the mathematics vocabulary terms presented at each grade level, we removed any duplicates and included 133 terms on the

initial measure (Powell et al., 2017).

To develop a measure exclusively for use with third grade, we noted the year of introduction for each term (e.g., K, 1, 2, or 3). We removed all terms introduced in grades 4, 5, or 6 to create Mathematics Vocabulary: Grade 3. This third-grade measure included 45 items in which students matched a term with a picture, provided a written response (e.g., wrote “5” for “Write an odd number.”), or drew a picture to represent a mathematics-vocabulary term (e.g., drew a triangle for “Draw a shape with three sides.”). Table 2 outlines each term and the grade level of introduction of the term. We noted textbook glossaries introduced 9 terms in kindergarten, 11 terms in first grade, 9 terms in second grade, and 16 terms in third grade. All 45 terms were included in third-grade glossaries indicating the importance of knowledge about these terms in third grade. Table 2 also provides a list of each of the 45 items with accuracy rates by difficulty and EL categories. After a pretest examiner provided directions, students had 12 min to answer as many questions as possible. Students received 1 point for each correct answer for a maximum score of 45. Cronbach’s α for Mathematics Vocabulary: Grade 3 was .92. We identified significant correlations between Single-Digit Word Problems and Mathematics Vocabulary: Grade 3 ($r = .610, p < .01$) and between Open Equations and Mathematics Vocabulary: Grade 3 ($r = .599, p < .01$). We also noted a significant correlation between Single-Digit Word Problems and Open Equations ($r = .539, p < .01$).

Scoring. Two examiners independently entered scores on 100% of the test protocols for each outcome measure on an item-by-item basis into an electronic database, resulting in two separate databases. We compared the discrepancies between the two databases across each outcome measure and rectified any inconsistencies to reflect the original response. Two examiners and the Project Manager resolved all discrepancies. Then, we converted students’

responses to correct (1) and incorrect (0) scores using spreadsheet commands, which ensured 100% accuracy of scoring. Original pretesting scoring reliability for 2016-2017 was 96.4% and 98.0% for 2017-2018, respectively.

Procedure and Examiners

In late August and early September, examiners participated in three, 3-hr pretesting trainings. Then, during the first week of September, we administered whole-class pretesting in one, 55-min session. In Cohort 1 (2016-2017), examiners ($n = 15$) conducted all of the pretesting sessions. Of the examiners, 13 were female and 2 were male, with 53.3% identifying as Caucasian ($n = 8$), 26.7% as Hispanic ($n = 4$), 13.3% as Asian American ($n = 2$), and 6.7% as African American ($n = 1$). In Cohort 2 (2017-2018), examiners ($n = 15$) also conducted all of the pretesting sessions. All Cohort 2 examiners were female, and 73% ($n = 11$) identified as Caucasian, 13.3% percent as Hispanic ($n = 2$), 6.7% as American Indian ($n = 1$), and 6.7% as African American ($n = 1$). In both cohorts, examiners were pursuing or had obtained a Master's or doctoral degree in an education-related field.

Data Analysis

We used ANOVA to identify differences among categories of students at pretest. Then, we conducted post-hoc pairwise comparisons with a Bonferroni correction to understand differences between conditions at pretest. We calculated effect sizes (ES) using Cohen's d .

Results

Comparison of ELs and Non-ELs

We conducted separate analyses comparing the Mathematics Vocabulary: Grade 3 performance of ELs and non-ELs in each of the four difficulty categories: no MD, EQD, WPD, or EQD+WPD (see Table 1). For students with no MD, non-ELs outperformed ELs, $F(1, 784) =$

93.05, $p < .001$. We noted a similar pattern for students with EQD, $F(1, 123) = 7.14, p = .009$. For students with WPD, however, there were no significant differences between ELs and non-ELs, $F(1, 132) = 2.34, p = .129$. The same was true for students with EQD+WPD, $F(1, 209) = 2.48, p = .117$.

Comparison Among Categories of Mathematics Difficulty

For ELs, we noted a significant difference on Mathematics Vocabulary: Grade 3 scores among the four categories of mathematics difficulty, $F(3, 509) = 62.153, p < .001$. Using post-hoc comparisons, we identified significant differences between students with no MD and EQD ($M_{\text{diff}} = 3.55, \text{CI} [1.13, 5.97], p = .001$), WPD ($M_{\text{diff}} = 5.08, \text{CI} [3.47, 6.70], p < .001$), and EQD+WPD ($M_{\text{diff}} = 7.09, \text{CI} [5.64, 8.54], p < .001$). We noted a significant difference between students with EQD and EQD+WPD ($M_{\text{diff}} = 3.54, \text{CI} [1.00, 6.08], p = .001$) as well as between students with WPD and EQD+WPD ($M_{\text{diff}} = 2.01, \text{CI} [0.22, 3.79], p = .018$).

We also compared the performance of non-ELs in the four categories: no MD, EQD, WPD, or EQD+WPD. We identified a significant difference on Mathematics Vocabulary: Grade 3 scores among the four conditions, $F(3, 739) = 84.703, p < .001$. Subsequently, we calculated significant differences between students with no MD and EQD ($M_{\text{diff}} = 5.47, \text{CI} [3.48, 7.45], p < .001$), WPD ($M_{\text{diff}} = 8.70, \text{CI} [5.64, 11.76], p < .001$), and EQD+WPD ($M_{\text{diff}} = 11.05, \text{CI} [8.92, 13.19], p < .001$). We also identified a significant difference between students with EQD and EQD+WPD ($M_{\text{diff}} = 5.58, \text{CI} [2.87, 8.30], p < .001$).

Mathematics-Vocabulary Terms

Table 2 provides accuracy rates, by mathematics-vocabulary term, for students in each category, and allows for investigation of performance patterns for individual terms by MD category and EL status. In this section, we highlight several trends of interest. In the area of

number and operations, students performed best with *even* and *odd*, better than any of the terms for components of addition, subtraction, multiplication, and division equations (e.g., *product* or *divisor*). For example, accuracy rates for *even* ranged from 0.51 for ELs with EQD+WPD to 0.94 for non-ELs with no MD. A similar pattern emerged for *odd*, with accuracy rates ranging from 0.50 for ELs with EQD+WPD to 0.95 for non-ELs with no MD. Students demonstrated greater understanding of terms related to place value (e.g., *standard form* and *expanded form*) than terms related to fractions. For instance, ELs and non-ELs across all four categories demonstrated accuracy rates ranging from 0.11 to 0.75 for *standard form*, compared to a range of 0.00 to 0.08 for *unit fraction*. Similarly, for *expanded form*, accuracy rates ranged from 0.21 to 0.80, compared to 0.08 to 0.39 for *fraction*.

For geometry-focused vocabulary, students performed extremely well on identification questions of *circle*, *rectangle*, *square*, and *triangle*, with accuracy rates ranging from 0.59 to 1.00 for ELs and non-ELs in all four categories. Students struggled more with other two-dimensional shapes, such as *rhombus* and *trapezoid*, as noted by the lower accuracy rate ranges of 0.25 to 0.59 for *rhombus* and 0.24 to 0.64 for *trapezoid*. For the language used to describe properties of two-dimensional shapes, such as *line* (range = 0.00 to 0.03) or *angle* (range = 0.00 to 0.10), students did not show high rates of success. Interestingly, students often performed better with the language used to describe three-dimensional shapes (e.g., *vertex*, *edge*, and *face*), with accuracy rates ranging from 0.15 to 0.74. Students also exhibited some success with names of three-dimensional shapes like *cube* (range = 0.21 to 0.79) and *triangular prism* (range = 0.10 to 0.59).

Finally, we asked students questions about mathematics vocabulary related to measurement and data. All students displayed superior understanding of *perimeter* over *area*,

with ranges from 0.20 to 0.54 for *perimeter*, compared to 0.00 to 0.19 for *area*. Students achieved similar rates of success for identification of *bar graph*, *pictograph*, and *tally chart*. For ELs and non-ELs, accuracy rates for graphing terms ranged from 0.01 to 0.54 across the four categories.

Overall, we noted very low accuracy rates for many mathematics-vocabulary terms, which was especially true for students with MD and ELs. For example, students with MD (either ELs or non-ELs) demonstrated accuracy rates of less than 10% on 16 terms (i.e., *angle*, *array*, *denominator*, *difference*, *dividend*, *divisor*, *factor*, *line*, *line segment*, *numerator*, *product*, *quotient*, *remainder*, *right angle*, *sum*, and *unit fraction*). For students, both ELs and non-ELs with EQD+WPD, we identified 24 terms (i.e., over half of the terms on the entire Mathematics Vocabulary: Grade 3 measure) with accuracy rates of less than 10% (add *addend*, *area*, *bar graph*, *equation*, *equivalent fractions*, *fraction*, *pictograph*, and *tally chart* to the list in the previous sentence).

Discussion

Performance Comparisons

We identified significant differences in mathematics-vocabulary performance for ELs and non-ELs categorized as having no MD ($ES = 0.78$) or experiencing EQD only ($ES = 0.54$), with non-ELs outperforming ELs. We expected performance differences between ELs and non-ELs given prior research demonstrating lower mathematics scores for ELs and the language demands of a mathematics-vocabulary measure (Orosco, 2014). However, we did not expect for non-ELs to continue to outperform ELs when students experienced WPD alone ($ES = 0.32$) or EQD+WPD ($ES = 0.24$). For students with MD in two areas (i.e., solving equations and solving word problems), the performance difference between ELs and non-ELs was small. Comorbid

MD (i.e., EQD+WPD) negatively influenced mathematics-vocabulary performance regardless if students were classified as ELs or non-ELs.

When we analyzed the data for ELs in the four categories of MD, we noticed a cascading pattern of performance. Students with no MD outperformed students with EQD ($ES = 0.65$), WPD ($ES = 0.98$), and EQD+WPD ($ES = 1.41$). When we compared students with EQD to students with WPD, there were no significant differences ($ES = 0.31$). However, when we compared EQD or WPD alone students to students with EQD+WPD, we identified significant differences ($ESs = 0.74$ and 0.45 , respectively). These results parallel those Forsyth and Powell (2017) discovered with a sample of fifth-grade students who answered questions about mathematics vocabulary. In that study, the authors compared students with no MD to those with MD, reading difficulty, or comorbid mathematics and reading difficulty. Students with comorbid difficulty demonstrated the weakest mathematics-vocabulary knowledge, students with no difficulty demonstrated the strongest mathematics-vocabulary cores, and students with only one type of MD performed similarly to one another.

For non-ELs, we noted a very similar pattern of results across the four MD categories. We calculated ESs of 0.84 , 1.57 , and 1.88 for students with no MD compared to students with EQD, WPD, and EQD+WPD. Students with EQD outperformed students with EQD+WPD ($ES = 1.06$). Although not significant, mathematics-vocabulary scores favored students with EQD over WPD ($ES = 0.66$) and WPD over EQD+WPD ($ES = 0.58$). This similar pattern of results for non-ELs and ELs based on MD category substantiated our results from our first research question. That is, students with comorbid difficulty demonstrated the least success with answering questions about mathematics vocabulary.

With regard to the accuracy rates by item, it is notable that many mathematics-vocabulary

terms appeared confusing for students. Even though students receive instruction on addition and subtraction as early as kindergarten (National Governors Association Center for Best Practices & Council of Chief State School Officers 2010) and many terms first appeared in textbook glossaries in kindergarten and first grade, many students demonstrated a lack of understanding of terms used to describe such operations (e.g., *addend*, *sum*, *difference*). Students performed better on terminology about many two-dimensional shapes (e.g., *triangle*, *rectangle*), also emphasized in kindergarten and first-grade curricula. In first grade, students identify three-dimensional figures, such as *rectangular prisms*, and we noticed this first-grade experience influence vocabulary about three-dimensional figures. Given that standards and textbooks emphasize *standard form* and *expanded form* in first and second grade, this earlier exposure transferred to some terms (e.g., *expanded form*). In first and second grade, students work with *pictographs* and *bar graphs*, yet we did not determine high rates of knowledge about such terms.

Overall, students without MD displayed low accuracy rates (i.e., below 50% accuracy) for over half of the 45 items on Mathematics Vocabulary: Grade 3. This result indicated that both ELs and non-ELs found questions about mathematics vocabulary demanding. We noted much lower accuracy rates for students with comorbid MD (i.e., difficulty with both equation solving and word-problem solving). That is, both ELs and non-ELs with comorbid difficulty only demonstrated accuracy rates above 50% for six items. Importantly, these six items were the same for ELs and non-ELs (*circle*, *even*, *odd*, *rectangle*, *square*, and *triangle*), and this finding illustrates that students with intense difficulty in the area of mathematics struggle with mathematics vocabulary, regardless of EL status.

Limitations

We administered only one measure of mathematics vocabulary, and this measure

prompted students to provide written (e.g., “5” or “ $3 + 4 = 7$ ”) or drawn responses to some of the questions. Even though the writing and drawing burden for students was low on the assessment, the reading demands may have presented a challenge for students with MD and/or ELs. Future research should consider utilizing a measure of mathematics vocabulary that allows students to provide oral responses and enables at-risk students to demonstrate mathematics knowledge without reading or writing. Future research also should investigate the reading and writing profiles of the students with MD to determine whether students with MD comorbid with reading or writing difficulties demonstrate differential performance to students with MD without other difficulties.

We timed the administration of Mathematics Vocabulary: Grade 3 to ensure the measure could be given during one whole-class assessment session. The timed assessment allowed information gathering about mathematics-vocabulary fluency but may have prevented an opportunity to collect data about mathematics-vocabulary accuracy. In an untimed session, students may have attempted to respond to a greater number of items. Based on feedback from the examiners, however, the majority of students finished the mathematics-vocabulary measure in under 12 min.

An additional limitation reflects the narrow scope of the pretesting assessments; we only administered one measure of equation solving and one measure of word-problem solving. One measure often is used for identification of MD (Nelson & Powell, 2018b), but future research should consider using multiple measures to confirm equation difficulty or word-problem difficulty. Our measure of equation solving focused exclusively on addition and subtraction. Similarly, our measure of word-problem solving required students to solve only one-step word problems featuring addition and subtraction. Future assessments should consider including items

related to multiplication, division, and multi-digit computation, as well as multi-step word problems.

Although our considerable sample size included over 1,200 third-grade students, all students attended elementary schools in the same school district in a single state. Also, the overwhelming majority of our ELs spoke Spanish as their native language. Future research should consider replication in other school districts in other states with a more diverse sample of ELs speaking languages other than Spanish.

Implications for Practice

Based on the results from our study, we recommend third-grade students receive instruction and practice on mathematics vocabulary. No student in our sample scored a perfect score on the mathematics-vocabulary measure. Therefore, teachers may need to provide explicit instruction about mathematics-vocabulary terms used in the classroom. Recommendations for teaching mathematics vocabulary include the following: using explicit instruction, teaching mnemonic strategies to remember vocabulary terms, building fluency through multiple exposures distributed across a unit or school year, using vocabulary grids to help students understand the similarities and differences among similar terms (e.g., *product* and *quotient*), creating flashcards of vocabulary terms, using storybooks to emphasize mathematics vocabulary, playing games about vocabulary, and using technology to practice vocabulary (Hassinger-Das, Jordan, & Dyson, 2015; Marin, 2018; Peterson-Brown et al., 2019; Riccomini, Smith, Hughes, & Fries, 2015; Zhao & Lapuk, 2019). As emphasized by Powell and Driver (2015), teachers need to explicitly introduce and practice mathematics vocabulary, especially when teachers provide mathematics instruction to ELs and students with MD. We stress that the research base for instruction about mathematics vocabulary is limited; future research needs to investigate which

instructional recommendations lead to improved mathematics-vocabulary knowledge, especially for students with MD and ELs. As researchers and teachers conduct empirical investigations of how to teach mathematics vocabulary, teachers may want to rely upon vocabulary-building strategies from the fields of reading and language arts (e.g., Coyne, Simmons, Kame'enui, & Stoolmiller, 2004; McKeown, Beck, Omanson, & Pople, 1985) with a focus on vocabulary strategies for students with learning difficulty (e.g., Ebbers & Denton, 2008; Taylor, Mraz, Nichols, Rickelman, & Wood, 2009) and ELs (e.g., Bolos, 2012; Vadasy, Sanders, & Nelson, 2015).

To ensure maximum exposure to mathematics-vocabulary terms outside of the mathematics classroom, teachers may want to create opportunities to teach mathematics-vocabulary content in science, social studies, and English language arts classes. Developing interdisciplinary lessons for students may offer a unique opportunity to expand students' access to and practice with mathematics-vocabulary terms. In third-grade science, for example, teachers may integrate math-vocabulary terms related to *three-dimensional shapes* to teach students to compare the structure of the Earth's surface using models. In third-grade social studies, teachers could include geometry terms within geography lessons. For example, teachers may instruct students to find the *area* of the state of Texas or the *perimeter* of the Washington Monument. In English language arts classrooms, teachers may ask students to create a *scatterplot* or *bar graph* to compare third-grade students' favorite afternoon snacks, and to write an explanation to describe the findings. Designing purposeful, interdisciplinary lessons that embed mathematics vocabulary further supports students in their understanding and application of essential terms across content areas.

Conclusion

Third-grade ELs demonstrated lower performance on a measure of mathematics vocabulary than their non-EL peers. However, when students exhibited difficulty with mathematics, especially word-problem solving, performance on the mathematics-vocabulary measure was comparable for ELs and non-ELs. Although all students are likely to benefit from focused instruction on the language of mathematics, ELs and students with MD require additional supports to understand and communicate using mathematics vocabulary.

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Table 1

Demographic Information and Means and Standard Deviations for Measures

Variable	No MD (<i>n</i> = 787)				Equation difficulty (<i>n</i> = 125)				Word problem difficulty (<i>n</i> = 135)				Word problem + Equation difficulty (<i>n</i> = 211)			
	EL (<i>n</i> = 242)		Non-EL (<i>n</i> = 545)		EL (<i>n</i> = 36)		Non-EL (<i>n</i> = 89)		EL (<i>n</i> = 101)		Non-EL (<i>n</i> = 34)		EL (<i>n</i> = 136)		Non-EL (<i>n</i> = 75)	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Female	117	48.3	261	47.9	25	69.4	45	50.6	39	38.6	23	67.7	76	55.9	44	58.7
Race																
African American	5	2.1	62	11.4	1	2.8	18	20.2	1	1.0	4	11.8	4	2.9	26	34.7
Asian American	13	5.4	27	5.0	1	2.8	3	3.4	2	2.0	1	2.9	6	4.4	1	1.3
Caucasian	7	2.9	307	56.3	1	2.8	43	48.3	2	2.0	6	17.6	1	0.7	9	12.0
Hispanic	207	8.6	86	15.8	29	80.6	14	15.7	95	94.0	18	52.9	114	83.8	34	45.3
Multi-racial	5	2.1	53	9.7	4	11.1	9	10.1	1	1.0	4	11.7	5	3.7	3	4.0
Other	5	2.1	10	1.8	0	0.0	2	2.2	0	0.0	1	2.9	6	4.4	2	1.3
School-identified disability	3	1.2	21	3.9	1	2.8	7	7.9	5	5.0	5	14.7	17	12.5	22	29.3
English learner	242	100.0	0	0.0	36	100.0	0	0.0	101	100.0	0	0.0	136	100.0	0	0.0
Retained	8	3.3	11	2.0	2	5.6	1	1.1	10	9.9	4	11.7	16	6.3	6	8.0
TELPAS Listening	3.16	0.76			3.03	0.80			2.50	0.97			2.30	0.93		
TELPAS Speaking	2.86	0.86			2.68	0.95			2.19	0.95			2.05	0.83		
TELPAS Reading	2.65	0.94			2.29	0.87			1.67	0.70			1.55	0.67		
TELPAS Writing	2.45	0.89			2.18	0.90			1.69	0.75			1.68	0.77		
TELPAS Composite	2.68	0.76			2.47	0.75			1.87	0.67			1.73	0.62		
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Single-Digit Word Problems	11.43	1.96	12.71	1.57	10.44	1.99	11.27	2.02	5.05	1.72	5.18	2.05	4.41	1.97	4.13	1.99
Open Equations	13.29	5.59	15.49	7.31	3.97	1.13	3.35	1.35	9.85	4.04	9.38	4.13	2.82	1.55	2.57	1.54
Mathematics Vocabulary: Grade 3	14.10	5.68	19.05	7.01	10.56	5.25	13.58	5.92	9.02	4.65	10.35	3.52	7.01	4.27	8.00	4.49

Note. EL = English learner; MD = Mathematics difficulty; TELPAS = Texas English Language Proficiency Assessment System.

Table 2
Accuracy Rates by Mathematics-Vocabulary Terms

Term	Grade introduced	No MD (<i>n</i> = 787)		Equation difficulty (<i>n</i> = 125)		Word problem difficulty (<i>n</i> = 135)		Word problem + Equation difficulty (<i>n</i> = 211)	
		EL (<i>n</i> = 242)	Non-EL (<i>n</i> = 545)	EL (<i>n</i> = 36)	Non-EL (<i>n</i> = 89)	EL (<i>n</i> = 101)	Non-EL (<i>n</i> = 34)	EL (<i>n</i> = 136)	Non-EL (<i>n</i> = 75)
addend	1	0.14	0.30	0.08	0.17	0.06	0.21	0.04	0.05
angle	3	0.03	0.10	0.03	0.03	0.00	0.03	0.00	0.01
area	2	0.08	0.19	0.06	0.11	0.01	0.00	0.00	0.00
array	3	0.05	0.12	0.06	0.00	0.03	0.00	0.01	0.01
bar graph	1	0.18	0.43	0.08	0.22	0.02	0.00	0.06	0.01
circle	K	0.99	0.99	1.00	0.99	0.93	1.00	0.91	0.96
cube	K	0.58	0.79	0.39	0.56	0.34	0.41	0.21	0.35
denominator	3	0.05	0.13	0.03	0.03	0.03	0.00	0.00	0.00
difference	K	0.08	0.11	0.00	0.02	0.00	0.09	0.00	0.00
dividend	3	0.08	0.12	0.03	0.08	0.01	0.06	0.02	0.03
divisor	3	0.07	0.11	0.00	0.07	0.02	0.03	0.02	0.03
edge	1	0.16	0.31	0.08	0.21	0.10	0.03	0.11	0.08
equation	3	0.19	0.54	0.06	0.27	0.06	0.12	0.01	0.07
equivalent fractions	3	0.09	0.14	0.06	0.10	0.08	0.06	0.04	0.07
even	2	0.85	0.94	0.72	0.84	0.64	0.68	0.51	0.56
expanded form	1	0.67	0.80	0.50	0.63	0.41	0.44	0.21	0.27
face	1	0.66	0.74	0.64	0.67	0.46	0.56	0.35	0.36
factor	3	0.03	0.07	0.00	0.01	0.01	0.03	0.01	0.03
fraction	2	0.35	0.39	0.17	0.20	0.11	0.26	0.09	0.08
line	3	0.01	0.01	0.00	0.00	0.03	0.03	0.00	0.00
line segment	3	0.10	0.08	0.03	0.07	0.10	0.09	0.07	0.01
numerator	3	0.05	0.13	0.06	0.01	0.03	0.00	0.01	0.00
odd	2	0.84	0.95	0.67	0.82	0.58	0.82	0.50	0.55

parallelogram	2	0.43	0.54	0.44	0.31	0.32	0.15	0.26	0.27
perimeter	3	0.37	0.54	0.25	0.39	0.36	0.26	0.21	0.20
pictograph	2	0.20	0.41	0.11	0.22	0.06	0.00	0.02	0.01
polygon	2	0.33	0.53	0.07	0.26	0.28	0.32	0.17	0.23
product	2	0.05	0.08	0.00	0.04	0.01	0.03	0.01	0.04
quadrilateral	2	0.31	0.40	0.08	0.13	0.14	0.15	0.05	0.08
quotient	3	0.01	0.03	0.00	0.01	0.00	0.03	0.00	0.00
rectangle	K	0.91	0.96	0.72	0.91	0.64	0.74	0.59	0.65
rectangular prism	1	0.35	0.59	0.22	0.36	0.14	0.12	0.10	0.16
remainder	3	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
rhombus	1	0.57	0.59	0.44	0.54	0.41	0.50	0.28	0.31
right angle	3	0.02	0.08	0.03	0.02	0.02	0.00	0.00	0.03
sides	K	0.25	0.49	0.14	0.24	0.11	0.26	0.05	0.11
square	K	0.93	0.96	0.89	0.94	0.75	0.97	0.68	0.76
standard form	1	0.50	0.75	0.25	0.51	0.17	0.32	0.11	0.13
sum	K	0.10	0.28	0.03	0.08	0.01	0.06	0.00	0.00
tally chart	1	0.26	0.54	0.11	0.25	0.07	0.03	0.03	0.04
trapezoid	1	0.58	0.64	0.50	0.54	0.41	0.24	0.31	0.33
triangle	K	0.98	0.98	0.97	0.98	0.82	0.94	0.73	0.83
triangular prism	1	0.39	0.59	0.22	0.38	0.14	0.15	0.10	0.20
unit fraction	3	0.05	0.08	0.03	0.01	0.00	0.03	0.01	0.03
vertex	K	0.27	0.48	0.22	0.31	0.15	0.21	0.15	0.08

Note. EL = English learner; K = Kindergarten; MD = Mathematics difficulty. This table provides accuracy rates, by mathematics-vocabulary term, for ELs and non-ELs within the following four categories: no MD, Equation difficulty, Word-problem difficulty, and Word-problem + Equation difficulty.